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MULTIFRACTAL BEHAVIOR OF "LIQUID WOOD" ON MULTIFRACTAL SPACE – TIME MANIFOLDS

BY

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Abstract. "Liquid wood" is a relatively new biocomposite material on the market, exhibiting a plethora of interesting properties and subsequent possible uses. Amongst experimentally determined properties, it has been established that "liquid wood" can be obtained in the guise of nanoparticles, suitable for 3D – printing, for example. However, 3D – printing requires both heat and pressure as a process. As such, when referring to "liquid wood" nanoparticles, physical phenomena, understood in the "classical" sense, do not abide by the "classical" rules anymore. Nevertheless, these phenomena may be analyzed using Fractal Theory of Motion. This article will attempt to explain several aspects of the behavior of "liquid wood", when analyzed at a nano – scale, on a space – time manifold.

Keywords: liquid wood; fractal; manifold; biocomposite.

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1. Introduction

The development of biocomposite materials is a societal response to the necessity of finding new, reliable and sustainable materials. This necessity stems from the need to curb pollution caused by the production of “classic” materials, plastics included.

It is a known fact that the first “modern” materials were considered to be what now encompasses an entire new class – that of composite materials. A composite material is made of matrix and reinforcement. Typical examples of composite materials are: Kevlar, Aramid, fiberglass, carbon fiber. The aeronautics industry was the prime user of such materials, followed by the automotive industry. Although possessing some remarkable qualities, the problem of composite materials is one shared with “classic” materials: they are not sustainable, as they require big amounts of energy to be produced and some of their constituents may include petroleum – derived products (Nägele *et al.*, 2013).

Thus, the next step is the development of biocomposites. Biocomposite materials are nothing else than bio-degradable composite materials. What this implies, is that the matrix is at least biodegradable and the reinforcement is made of natural fibers (Pilla, 2011).

Biocomposites have important advantages: they are renewable, biodegradable and recyclable. They can be used alone or in combination with other composites. They pollute significantly less, are relatively easy to work with, and can be made into a wide variety of shapes and products.

“Liquid wood” is actually a polymeric biocomposite material, first developed by a German company, Tecnaro GmbH. In the case of “liquid wood”, the matrix is made of lignin and the reinforcement is made of various natural fibers. The “polymeric” aspect refers to the material’s behavior, which is akin to thermoplastics – it can be injected and turned into various objects. The material comes into three major presentation forms: Arboform, Arbofill and Arboblend, with different strengths and workability properties. Arboblend, for example, has been proven to be the easiest to work with, requiring the lowest temperatures and pressures to be brought into an injectable form. It is highly resistant to acids and alkalis. This suggests it might be used, for example, in the medicine field (Tecnaro GmbH, 2019).

As such, “liquid wood” seems to be a good candidate for a modern, sustainable material with a good range of applications.

Being a biocomposite system, “liquid wood” dynamics can be assimilated to a multifractal so, in the following, its behavior when subjected to different constraints, will be analyzed.

2. Biocomposites "Mimed" as Multifractals

Chaocity and non-linearity are of both functional and structural nature for any biocomposite. Interactions between their constitutive entities lead to reciprocal constraints and micro/macroscopic, individual/collective and local/global behavior types. In such a context, the universality of laws for biocomposite, turns out to be natural and has to be emulated in mathematical processes. These take the shape of diverse theoretical models that are able todepict their dynamics (Mitchell, 2009; Nottale, 2011).

Classical, regularly employed models are typically established, on the rather gratuitous hypothesis, that variables depicting the dynamics of biocomposites are differentiable. Therefore, the accomplishment of previously – mentioned models has to be seen as gradual/sequential, on domains where differentiability and integrability nonetheless hold true. The differentiable and integrable mathematical procedures are elseways lacking, when the dynamics of biocomposites should be worked out, dynamics that include both non-linearity and chaoticity. Nevertheless, to be able to characterize the aforementioned dynamics, whilst still making use of differential mathematical processes, it is imperative to pointedly present the scale resolution in the expression of variables associated with biocomposite dynamics. Essentially, it should be found in the expression of fundamental equations that dictate those dynamics. Hence, any variable dependent on space – time coordinates, in a classical sense, will rely on scale resolution within the afresh mathematical sense (the one of non-differentiability and non-integrability). Putting it differently, rather than working with a variable illustrated through a non-differentiable function, estimates of said mathematical function, acquired by mediation at diverse scale resolutions will be undertaken. For this reason, any physical variable developed to portray biocomposite dynamics, will function as the boundary of a group of mathematical functions, being non-differentiable for zero scale resolutions and differentiable for non- zero scale resolutions (Mitchell, 2009; Nottale, 2011).

This method of describing biocomposite dynamics distinctly asserts the establishment of a new geometrical structure and more so, of a novel theory for biocomposites. In said method, the motion laws, invariant to spatial and temporal transformations, are integrated with scale laws, invariant to spatial and temporal scales transformations. In the author's belief, this kind of geometrical structure may be anchored in the concept of a "multifractal", and the matching mathematical model may be supported by the The Scale Relativity Theory in an arbitrary and constant fractal dimension (Nottale, 2011).

The fundamental assumption of the proposed model is that biocomposite particle dynamics will be outlined by continuous but non-differentiable motion curves (multifractal motion curves) (Nottale, 2011). Such multifractal motion curves exhibit the trait of self – similarity in every point, that may be converted into a property of holography (every part reflects the whole). Basically, the discussion revolves around “holographic implementations of biocomposite particles dynamics” either through Schrödinger – type multifractal “regimes” (*i.e.* describing biocomposite particles dynamics by making use of Schrödinger – type equations at various scale resolutions) or trough hydrodynamic – type multifractal “regimes” (*i.e.* describing biocomposite particles dynamics by using hydrodynamic – type equation at various scale resolutions).

3. Time as a Multifractal

Scrutinizing the non-relativistic dynamics of all complex systems in a multifractal space, it is probable to notice a disparity amongst the space coordinates and the temporal one (regarded as an affine parameter of motion curve). As long as the space coordinates are multifractal, the temporal coordinate is shall not be a multifractal. This disparity has, as a consequence, the fact that any biocomposite entities travel on an infinite length curve (*i.e*. a multifractal curve), in a finite time interval. At a first glance, it would result that the entities have an infinite velocity. To do away with this paradox, in the following, it is assumed that not just the space coordinates are multifractal, but what is more, the temporal one is a multifractal. Essentially, the dynamics of any biocomposite will be constructed on multifractal space – time manifolds. In these frameworks, the most significant elements from the non-relativistic approach of Multifractal Theories of Motion, hold true, however the time differential element *dt* is substituted with the adequate time differential element $d\tau$. Thus, not just the space, but rather the entire space – time continuum is viewed as a multifractal.

4. Consequences of Non-Differentiability on a Space – Time Manifold

Assume that on a space – time manifold, the motions of all biocomposite entities happen on continuous but non-differentiable curves (multifractal curves). The non-differentiability of motion curves point out the next:

i) Any continuous but non-differentiable curve is notably scale dependent *i.e.* its length leans toward infinity when its appropriate time interval, $\Delta\tau$, tends to zero. This culminates in a natural development of the Lebesgue theorem on a space – time manifold. Because of this, in said limit, a curve in a space – time manifold is zig – zagged, as one can deduce. It displays the attribute of self–similarity in all its points of a space – time manifold, that maybe equated into an extension property of holography (every part of a space – time manifold reflects the entirety of the same space – time manifold).

To sum up, a continuous but non-differentiable space – time manifold is multifractal in the Mandelbrot's sense (Mandelbrot, 1983);

ii) The differential appropriate time reflection invariance of any variable is broken. For instance, the adequate time derivative of 4-coordinate X^{μ}; $\mu = 0,1,2,3$ takes the following form:

$$\left[\frac{dX^{\mu}}{d\tau}\right]_{+} = \lim_{\Delta\tau \to 0_{+}} \frac{X^{\mu}(\tau + \Delta\tau) - X^{\mu}(\tau)}{\Delta\tau}$$
$$\left[\frac{dX^{\mu}}{d\tau}\right]_{-} = \lim_{\Delta\tau \to 0_{-}} \frac{X^{\mu}(\tau) - X^{\mu}(\tau - \Delta\tau)}{\Delta\tau} \tag{1}$$

These relations are similar in the differentiable situation, $\Delta\tau \to -\Delta\tau$. In the non-differentiable matter, the prior definitions fall short, considering that the limits $\Delta\tau \to 0\pm$, are not defined anymore. Viewed through the Multifractal Theory of Motion, the physical phenomena are akin to the demeanor of the function throughout the "zoom" operation on the suitable time resolution $\delta\tau$: So, by virtue of the substitution process, $\delta\tau$ will be equated with the differential element $d\tau$, *i.e.*, $\delta\tau \equiv d\tau$. Hence, every differential variable $Q(\tau)$ is supplanted by the non-differentiable variable $Q(\tau, d\tau)$, expressly reliant on the proper time resolution interval, whose derivative is non-defined only in the limit, $\Delta\tau \to 0$. Consequently, a pair of derivatives of every non-differentiable variable as explicit functions of τ and $d\tau$, are to be specified. For instance, the two derivatives of the 4-coordinate $X^{\mu}(\tau, \Delta\tau)$ take the following forms:

$$\frac{d_{+}X^{\mu}}{d\tau} = \lim_{\Delta\tau \to 0_{+}} \frac{X^{\mu}(\tau + \Delta\tau, \Delta\tau) - X^{\mu}(\tau, \Delta\tau)}{\Delta\tau}$$
$$\frac{d_{-}X^{\mu}}{d\tau} = \lim_{\Delta\tau \to 0_{-}} \frac{X^{\mu}(\tau, \Delta\tau) - X^{\mu}(\tau - \Delta\tau, \Delta\tau)}{\Delta\tau} \tag{2}$$

The sign "+" represents the forward physical process and the sign "–" conforms to the rearward one;

iii) The differential of 4-coordinate $dX^{\mu}(\tau, \Delta\tau)$ may be stated as the sum of two differentials, one not being scale dependent (differentiable part $d_{\pm}x^{\mu}(\tau)$) and the other being scale dependent (non-differentiable part $d_{\pm}\xi^{\mu}(\tau, d\tau)$),

$$d_{\pm}X^{\mu}(\tau, \Delta\tau) = d_{\pm}x^{\mu}(\tau) + d_{\pm}\xi^{\mu}(\tau, \Delta\tau) \tag{3}$$

iv) The non-differentiable part of the 4-coordinate ξ^{μ} fulfills the non-differentiable equation

$$d_{\pm}\,\xi^{\mu}(\tau,\Delta\tau)=\lambda_{\pm}^{\mu}(d\tau)^{1/D_F} \tag{4a}$$

where $\lambda_{\pm}^{\mu}$ are constant coefficients associated to differential – nondifferential transition, whose statistical sense will be made known in what follows and D_F is the fractal dimension of the motion curves from the space – time manifold.

In the author's opinion, the non-differentiable part of 4-coordinate$d\ X^{\mu}$ fulfills the multi – fractal equation:

$$d_{\pm}\xi^{i}(t,dt)=\lambda_{\pm}^{i}(dt)^{\left[2/f(\alpha)\right]-1},\qquad \alpha=\alpha(D_F) \tag{4b}$$

$f(\alpha)$ is the singularity spectrum of order α and α is a singularity index. There are many modalities, and thus a varied selection of definitions of fractal dimensions: more precisely, the fractal dimension in the sense of Kolmogorov, the fractal dimension in the sense of Hausdorff–Besikovitch etc. (Nottale, 2011; Mandelbrot 1983). Selecting one of these definitions and operating it in the biocomposite multifractal dynamics, the value of the fractal dimension must be constant and arbitrary for the entirety of the dynamical analysis: for example, regular finds can be $D_F<2$ for correlative processes, $D_F>2$ for non-correlating processes etc.

In such a conjecture, through (4b) it is possible to identify not only the "areas" of the atmospheric multifractal that are characterized by a certain fractal dimension, but also the number of "areas" whose fractal dimensions are situated in an interval of values. More than that, through the singularity spectrum $f(\alpha)$ it is possible to identify classes of universality in the biocomposite dynamics laws, even when regular or strange attractors have different aspects (Cristescu, 2008).

v) The differential suitable time reflection invariance is salvaged by joining the derivatives $d_+/\,d\tau$ and $d_-/\,d\tau$ in the non-differentiable operator

$$\frac{\hat{d}}{d\tau}=\frac{1}{2}\left(\frac{d_{+}+d_{-}}{d\tau}\right)-\frac{i}{2}\left(\frac{d_{+}-d_{-}}{d\tau}\right) \tag{5}$$

This operation, as specified by Cresson (Cresson, 2000), is labeled "differentiability by extension in complex on a space-time manifold" (Cresson's theorem). Using the non-differentiable operator to the 4-coordinate X^{μ} yields the complex velocity:

$$\hat{V}^{\mu}=\frac{\hat{d}X^{\mu}}{d\tau}=\frac{1}{2}\left(\frac{d_{+}X^{\mu}+d_{-}X^{\mu}}{d\tau}\right)-\frac{i}{2}\left(\frac{d_{+}X^{\mu}-d_{-}X^{\mu}}{d\tau}\right) \tag{6}$$

with

$$V^{\mu} = \frac{1}{2}\left(v_{+}^{\mu} + v_{-}^{\mu}\right), U^{\mu} = \frac{1}{2}\left(v_{+}^{\mu} - v_{-}^{\mu}\right),$$
$$v_{+}^{\mu} = \frac{d_{+}x^{\mu} + d_{+}\xi^{\mu}}{d\tau}, v_{-}^{\mu} = \frac{d_{-}x^{\mu} + d_{-}\xi^{\mu}}{d\tau} \tag{7}$$

The real part V^{μ} is differentiable and scale resolution independent, while the imaginary one U^{μ} is non-differentiable and scale resolution dependent;

vi) An infinite number of geodesics may be discovered, relating to any pair of points of a space – time manifold. This is true on the entirety of scale resolutions of the dynamics of complex systems. So, in the space – time manifold, the entire biocomposite entities are replaced with the geodesics themselves. As such, any external constraint can be explained as a range of geodesics in the same space – time manifold. The infinity of geodesics in the bundle, their non-differentiability, the two values of the derivative etc., hint towards a generalized statistical fluid – like description (multifractal fluid). In this manner, the type of multifractalization can be provided with the contribution of stochastic processes. In light of this, averages, variances, covariances *i.e.* the entire arsenal of statistics of the multifractal fluid variables, has to be taken into account, in the sense of the stochastic process correlated to multifractalization. In this regard, the choice of the average of $d_{\pm}X^{i}$ in the form

$$\langle d_{\pm}X^{i}\rangle \equiv d_{\pm}x^{i} \tag{8}$$

implies through (3)

$$\langle d_{\pm}\xi^{i}\rangle = 0 \tag{9}$$

5. Motion Non-Differentiable Operator on a Space – Time Manifold

Let it be considered that the movement curves (continuous and non-differentiable) are immersed in space – time and that X^{μ} are the 4-coordinates of a point on the curve. Moreover, taking into account a variable $Q(X^{\mu}, \tau)$ and the subsequent Taylor expansion, up to the second order (a reminder that, through Cresson's theorem, the multifractal function becomes differentiable in any point of the space – time manifold)

$$d_{\pm}Q(X^{\mu}, \tau, d\tau) = \partial_{\tau}Q d\tau + \partial_{\mu}Q d_{\pm}X^{\mu} + \frac{1}{2}\partial_{\mu}\partial_{\nu}Q d_{\pm}X^{\mu} d_{\pm}X^{\nu} \tag{10}$$

where

$$\partial_\tau = \frac{\partial}{\partial_\tau},\ \partial_\mu = \frac{\partial}{\partial X^\mu}, \partial_\mu \partial_\nu = \frac{\partial^2}{\partial X^\mu \partial X^\nu}$$

Relations (10) remain true in any point of the space – time manifold and more so for the points "X^μ" on the non-differentiable curve which were picked in relation (10).

Henceforth, forward and rearward average values of (10) turn into

$$\langle d_\pm Q(X^\mu, \tau, d\tau)\rangle = \langle \partial_\tau Q d\tau\rangle + \langle \partial_\mu Q d_\pm X^\mu\rangle + \frac{1}{2}\partial_\mu \partial_\nu Q \langle d_\pm X^\mu d_\pm X^\nu\rangle \qquad (11)$$

The ensuing provisions are made: the median values of the variables $Q(X^\mu, \tau, d\tau)$ and its derivatives match with themselves and additionally, the differentials $d_\pm X^\mu$ and $d\tau$ are independent. Accordingly, the median of their products coincides with the product of their medians averages. In such circumstances, (11) take the shape

$$d_\pm Q(X^\mu, \tau, d\tau) = \partial_\tau Q d\tau + \partial_\mu Q \langle d_\pm X^\mu\rangle + \frac{1}{2}\partial_\mu \partial_\nu Q \langle d_\pm X^\mu d_\pm X^\nu\rangle \qquad (12)$$

or, using (3), (8) and (9)

$$\begin{aligned} d_\pm Q(X^\mu, \tau, d\tau) &= \partial_\tau Q d\tau + \partial_\mu Q d_\pm x^\mu + \\ &+ \frac{1}{2}\partial_\mu \partial_\nu Q (d_\pm x^\mu d_\pm x^\nu + \langle d_\pm \xi^\mu d_\pm \xi^\nu\rangle) \end{aligned} \qquad (13)$$

Although the average values of the 4-non-differentiable coordinate $d_\pm \xi^\mu$ is null, for the higher order of the 4-non-differentiable coordinate average, the situation may be distinct. Fixating on the mean $< d_\pm \xi^\mu d_\pm \xi^\nu >$ and applying (4) it is possible to write

$$\langle d_\pm \xi^\mu d_\pm \xi^\nu\rangle = \pm \lambda_\pm^\mu \lambda_\pm^\nu (d\tau)^{(2/D_F - 1)} d\tau \qquad (14)$$

using the convention that the sign "+" equates to $d\tau > 0$, while the sign "–" matches to $d\tau < 0$.

Then (13) takes the form

$$\begin{aligned} d_\pm Q(X^\mu, \tau, d\tau) = \partial_\tau Q d\tau + \partial_\mu Q d_\pm x^\mu + \frac{1}{2}\partial_\mu \partial_\nu Q d_\pm x^\mu d_\pm x^\nu \\ \pm \frac{1}{2}\partial_\mu \partial_\nu Q \lambda_\pm^\mu \lambda_\pm^\nu (d\tau)^{(2/D_F - 1)} d\tau \end{aligned} \qquad (15)$$

Dividing by $d\tau$ and discounting the terms that include differential factors, employing the process from (Nottale, 2011; Mercheș and Agop, 2016) it is obtained:

$$\begin{gathered}\frac{d_{\pm}Q(X^{\mu},\tau,d\tau)}{d\tau}=\partial_{\tau}Q+v_{\pm}^{\mu}\partial_{\mu}Q\pm\\\pm\frac{1}{2}\lambda_{\pm}^{\mu}\lambda_{\pm}^{\nu}(d\tau)^{(2/D_{F}-1)}\partial_{\mu}\partial_{\nu}Q\end{gathered}\tag{16}$$

Furthermore, these relations permit to define the operators:

$$\frac{d_{\pm}}{d\tau}=\partial_{\tau}+v_{\pm}^{\mu}\partial_{\mu}\pm\frac{1}{2}\lambda_{\pm}^{\mu}\lambda_{\pm}^{\nu}(d\tau)^{(2/D_{F}-1)}\partial_{\mu}\partial_{\nu}\tag{17}$$

Given these circumstances, one can compute $\hat{d}/d\tau$. Factoring in (5), (6) and (17), it is obtained:

$$\begin{gathered}\frac{\hat{d}Q}{d\tau}=\frac{1}{2}\left[\left(\frac{d_{+}Q+d_{-}Q}{d\tau}\right)-i\left(\frac{d_{+}Q-d_{-}Q}{d\tau}\right)\right]=\\\partial_{\tau}Q+\hat{V}^{\mu}\partial_{\mu}Q+\frac{1}{4}(d\tau)^{(2/D_{F}-1)}D^{\mu\nu}\partial_{\mu}\partial_{\nu}Q\end{gathered}\tag{18}$$

where

$$\begin{gathered}D^{\mu\nu}=d^{\mu\nu}-i\bar{d}^{\mu\nu}\\d^{\mu\nu}=\lambda_{+}^{\mu}\lambda_{+}^{\nu}-\lambda_{-}^{\mu}\lambda_{-}^{\nu},\bar{d}^{\mu\nu}=\lambda_{+}^{\mu}\lambda_{+}^{\nu}+\lambda_{-}^{\mu}\lambda_{-}^{\nu},\qquad i=\sqrt{-1}\end{gathered}\tag{19}$$

The relation additionally allows defining the motion non-differentiable operator

$$\frac{\hat{d}}{d\tau}=\partial_{\tau}+\hat{V}^{\mu}\partial_{\mu}+\frac{1}{4}(d\tau)^{(2/D_{F}-1)}D^{\mu\nu}\partial_{\mu}\partial_{\nu}\tag{20}$$

If the non-differentiability of motion curves is carried out with the help of Markov type stochastic process (Nottale, 2011; Mandelbrot, 1983)

$$\lambda_{+}^{\mu}\lambda_{+}^{\nu}=\lambda_{-}^{\mu}\lambda_{-}^{\nu}=-\lambda\eta^{\mu\nu}\tag{21}$$

where $\eta^{\mu\nu}$ is the Minkowski metric and λ is the coefficient related to the differentiable – nondifferentiable transition, then the motion non-differentiable operator has the shape:

$$\frac{\hat{d}}{d\tau}=\partial_{\tau}+\hat{V}^{\mu}\partial_{\mu}+i\frac{\lambda}{2}(d\tau)^{(2/D_{F}-1)}\partial_{\mu}\partial^{\mu}\tag{22}$$

If the non-differentiability of motion curves is carried out through non-Markov type stochastic process (Nottale, 2011; Mandelbrot, 1983)

$$\lambda_+^\mu \lambda_+^\nu - \lambda_-^\mu \lambda_-^\nu = \lambda_1 \eta^{\mu\nu}$$
$$\lambda_+^\mu \lambda_+^\nu + \lambda_-^\mu \lambda_-^\nu = -\lambda_2 \eta^{\mu\nu} \tag{23}$$

where λ_1 and λ_2 are two coefficients associated to the differentiable – nondifferentiable transition, then the motion non-differentiable operator has the outcome:

$$\frac{\hat{d}}{d\tau} = \partial_\tau + \hat{V}^\mu \partial_\mu + \frac{1}{4}(\lambda_1 + i\lambda_2)(d\tau)^{(2/D_F-1)} \partial_\mu \partial^\mu \tag{24}$$

6. Scale Covariance Derivative. Non-differentiable Geodesics on a Space – Time Manifold

In this section, let it be examined the functionality of the scale covariance principle (Nottale, 2011): the physics laws are simultaneously invariant when referring to the 4 – coordinates transformation and regarding scale transformations. Thus the passage from differentiable physics in a space – time manifold to the non-differentiable physics (*i.e.* the Multifractal Theory of Motion) in the same space – time manifold that is to be considered, can be put into effect by swapping the standard derivative $d/d\tau$ with the non-differentiable operator $\hat{d}/d\tau$. Consequently, this operator takes the role of a "scale covariant derivative" operator, specifically, being utilized to type the fundamental equations of dynamics of biocomposite under the alike shape as in the differentiable case. As such, implementing the operator (20) in the complex velocity (6), the geodesics equation turns into:

$$\frac{\hat{d}\hat{V}^\mu}{d\tau} = \partial_\tau \hat{V}^\mu + \hat{V}^\nu \partial_\nu \hat{V}^\mu + \frac{1}{4}(d\tau)^{(2/D_F-1)} D^{\alpha\beta} \partial_\alpha \partial_\beta \hat{V}^\mu \equiv 0 \tag{25}$$

or, also making use of (6), by splitting the motions on scale resolutions (the real part from the imaginary one)

$$\frac{\hat{d}V^\mu}{d\tau} = \partial_\tau V^\mu + V^\nu \partial_\nu V^\mu - U^\nu \partial_\nu U^\mu + \frac{1}{4}(d\tau)^{(2/D_F-1)} d^{\alpha\beta} \partial_\alpha \partial_\beta V^\mu - \frac{1}{4}(d\tau)^{(2/D_F-1)} \bar{d}^{\alpha\beta} \partial_\alpha \partial_\beta U^\mu = 0$$
$$\frac{\hat{d}U^\mu}{d\tau} = \partial_\tau U^\mu + V^\nu \partial_\nu U^\mu + U^\nu \partial_\nu V^\mu + \frac{1}{4}(d\tau)^{(2/D_F-1)} d^{\alpha\beta} \partial_\alpha \partial_\beta U^\mu + \frac{1}{4}(d\tau)^{(2/D_F-1)} \bar{d}^{\alpha\beta} \partial_\alpha \partial_\beta V^\mu = 0 \tag{26}$$

For motions on non-differentiable curves realized by Markov type stochastic process, the geodesics equation becomes

$$\frac{\hat{d}\hat{V}^{\mu}}{d\tau} = \partial_{\tau}\hat{V}^{\mu} + \hat{V}^{\nu}\partial_{\nu}\hat{V}^{\mu} - i\frac{\lambda}{2}(d\tau)^{(2/D_F-1)}\partial^{\nu}\partial_{\nu}\hat{V}^{\mu} = 0 \tag{27}$$

or, through separation of motions on scale resolutions

$$\begin{aligned} \frac{\hat{d}V^{\mu}}{d\tau} &= \partial_{\tau}V^{\mu} + V^{\nu}\partial_{\nu}V^{\mu} - \left(U^{\nu} - \frac{\lambda}{2}(d\tau)^{(2/D_F-1)}\partial^{\nu}\right)\partial_{\nu}U^{\mu} = 0 \\ \frac{\hat{d}U^{\mu}}{d\tau} &= \partial_{\tau}U^{\mu} + V^{\nu}\partial_{\nu}U^{\mu} + \left(U^{\nu} - \frac{\lambda}{2}(d\tau)^{(2/D_F-1)}\partial^{\nu}\right)\partial_{\nu}V^{\mu} = 0 \end{aligned} \tag{28}$$

The first relation (26) contains the expression of 4-multifractal specific force.

For motions on non-differentiable curves realized by non-Markov type stochastic process, the geodesics equation becomes

$$\frac{\hat{d}\hat{V}^{\mu}}{d\tau} = \partial_{\tau}\hat{V}^{\mu} + \hat{V}^{\nu}\partial_{\nu}\hat{V}^{\mu} + \frac{1}{4}(\lambda_1 + i\lambda_2)^{(2/D_F-1)}\partial_{\nu}\partial^{\nu}\hat{V}^{\mu} = 0 \tag{29}$$

or, by splitting the motions on scale resolutions

$$\begin{aligned} \frac{\hat{d}V^{\mu}}{d\tau} &= \partial_{\tau}V^{\mu} + V^{\nu}\partial_{\nu}V^{\mu} - \left(U^{\nu} - \frac{\lambda_2}{4}(d\tau)^{(2/D_F-1)}\partial^{\nu}\right)\partial_{\nu}U^{\mu} \\ &\quad + \frac{\lambda_1}{4}(d\tau)^{(2/D_F-1)}\partial_{\nu}\partial^{\nu}V^{\mu} = 0 \\ \frac{\hat{d}U^{\mu}}{d\tau} &= \partial_{\tau}U^{\mu} + V^{\nu}\partial_{\nu}U^{\mu} + \left(U^{\nu} - \frac{\lambda_2}{4}(d\tau)^{(2/D_F-1)}\partial^{\nu}\right)\partial_{\nu}V^{\mu} \\ &\quad + \frac{\lambda_1}{4}(d\tau)^{(2/D_F-1)}\partial_{\nu}\partial^{\nu}U^{\mu} = 0 \end{aligned} \tag{30}$$

Let it be noted, in the non-relativistic approximation of motion, the operator (22) becomes:

$$\frac{\hat{d}}{dt} = \partial t + \hat{V}^{l}\partial_{e} + \frac{1}{4}(dt)^{\left(\frac{2}{D_f}\right)^{-1}} D^{ek}\partial_{e}\partial_{k} \tag{31}$$

whilst the complex speed (6) takes the form:

$$\hat{V}^i = V_D^i - iV_F \, , \; i = 1, 2, 3 \qquad (32)$$

In (31), V_D^i is the differentiated speed, independent of the scale resolution *dt* and V_F is the differentiated speed, dependent of the same scale resolution.

7. Conclusions

The main conclusions of the presented paper are the following:

i) A discussion has been made, concerning the assimilation of any biostructure to a multifractal, based on its dynamic behaviors.

ii) On a space – time manifold, the main characteristics of non-differentiability were specified, in the case of biostructure dynamics.

iii) A motion operator was built and the geodesics (motion equations) in the space – time manifold were established.

iv) Based on a scale covariance principle, the motion equations were explained, both for global scale resolution as well as for the separation of global scale resolution (both on differentiable scale, as well as on non-differentiable scale).

v) The operation with 4-dimensional space – time manifolds on the biostructure dynamics, has the advantage of allowing the simultaneous definition of both a time and a critical velocity, specific to every biostructure (more precisely, the biological time - $\Delta\tau$ of a biostructure and the corresponding growth speed).

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COMPORTAMENT MULTIFRACTAL AL "LEMNULUI LICHID" PE VARIETĂȚI SPAȚIU – TIMP MULTIFRACTALE

(Rezumat)

"Lemnul lichid" este un biocompozit relativ nou, având o serie de proprietăți interesante și posibile utilizări. Printre proprietățile determinate experimental, s-a stabilit faptul ca "lemnul lichid" poate fi obținut sub formă de nanoparticule, potrivite, spre exemplu, pentru printare 3D. Totuși, printarea 3D utilizează căldură și presiune, ca procese tehnologice. Așadar, când se face referire la nanoparticulele de "lemn lichid", fenomenele fizice, înțelese în sens "clasic", nu se mai supun regulilor "clasice". Cu toate acestea, aceste fenomene pot fi analizate utilizând Teoria Fractală a Mişcării. Acest articol caută să explice câteva aspecte ale comportamentului "lemnului lichid", atunci când este analizat la nano – scară, pe varietate spațiu – timp.